Confinement, Chiral Symmetry Breaking and Faddeev-Niemi Decomposition in QCD

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ABSTRACT: We identify two distinct, complementary classes of gauge field configurations for QCD with $SU(2)$ gauge group, one (*instanton-like configurations*) having to do with chiral symmetry breaking but not with confinement, the other (*regularized Wu-Yang monopoles*) very likely responsible for confinement but unrelated to chiral symmetry breaking. Our argument is based on a semiclassical analysis of fermion zero modes in these backgrounds, made by use of a gauge field decomposition recently introduced by Faddeev and Niemi. Our result suggests that the two principal dynamical phenomena in QCD, confinement and chiral symmetry breaking, are distinct effects, caused by two competing classes of gauge field configurations.

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It is now widely accepted that confinement in Quantum Chromodynamics (QCD) is a kind of dual superconductivity [1, 2, 3]. According to this idea, the ground state of QCD is a condensate of magnetic monopoles. Color electric fields are expelled from the hadronic medium, except as possible thin filaments (Abrikosov votex) connecting quarks and antiquarks, leading to a linearly rising potential between them.

The details of this phenomen is difficult to analyse, though, due to the fact that the role of the Cooper pairs in the standard superconductors is here played by topologically nontrivial soliton-like configurations. Actually, the situation is worse since, unlike what happens in theories with elementary scalar fields in the adjoint representation, magnetic monopoles in QCD do not correspond to any stable solitonic solutions of the classical Yang-Mills equations of motion. In fact such a static solution is known not to exist in pure Yang-Mills theories. Rather they represent sets of regularized Wu-Yang monopole configurations, whose presence may be conveniently detected by 't Hooft's Abelianization procedure [2].

A new decomposition of the Non-Abelian gauge fields recently proposed by Faddeev and Niemi [4, 5] makes these ideas more concrete; it appears to enables us to analyse the relevant physics aspects in more detail, without the need of choosing particular gauges such as the maximally Abelian gauge [6].

As an example, we discuss here the possible connection between confinement and chiral symmetry breaking in QCD. We shall argue that configurations responsible for confinement and for chiral symmetry breaking are quite distinct and in a sense complementary, in QCD.

A key aspect in discussing such a connection is the existence of the fermion zero modes in the background of the semiclassical monopoles [7]. In fact, the connection between these two dynamical phenomena has been recently illucidated in full nonperturbative analysis, in many $SU(n_c)$ or $USp(2n_c)$ theories with $N = 2$ supersymmetry (broken softly to $N = 1$) [8, 9]. We shall not discuss here physics of these models (which show very rich varieties of nonperturbative scenarios, some resembling those in the actual world of strong interactions, some quite different); for the present purpose we shall only draw one lesson from these studies. Namely the existence of the fermion zero modes in the background of semiclassical monopoles, is a necessary condition for the low-energy, light magnetic monopoles to carry flavor quantum numbers, hence a condition for their condensation (confinement) to imply chiral symmetry breaking.

The models of [8, 9] contain 't Hooft-Polyakov monopoles [10] in the spectrum.

The existence and number of the zero modes for each fermion can be (and has been) established both explicitly (by generalizing the analysis by Jackiw and Rebbi) and through Callias' index theorem [11]. The resulting semiclassical flavor multiplet structure can be compared with the spectrum of low-energy massless (fully quantum mechanical) monopoles, with complete matching between the two [9].

In QCD, one might instead use the background configurations which are presumably dominant in its ground state [12, 13]. For simplicity, we discuss here QCD with $SU(2)$ gauge group. Candidate configurations we shall consider are (regularized) Wu-Yang monopoles, instantons, and modification/collection of these. According to Faddeev and Niemi, the $SU(2)$ connection can be decomposed as

$$A_\mu^a = C_\mu \mathbf{n}^a + \tilde{\sigma}(x)(\partial_\mu \mathbf{n} \times \mathbf{n})^a + \rho\, \partial_\mu \mathbf{n}^a; \qquad \tilde{\sigma}(x) = 1 + \sigma(x), \tag{1}$$

in terms of the unit vector field $\mathbf{n}$ and the Abelian gauge field C_μ which live on S^2 and S^1 factors, respectively, of $SU(2) = S^3 \sim S^2 \times S^1$ manifold, and a charged "scalar" field

$$\phi = \rho(x) + i\sigma(x). \tag{2}$$

In terms of these variables, the Wu-Yang singular monopole solution [14], for instance, is

$$n^a = \frac{x^a}{r}, \quad C_\mu = \phi = 0. \tag{3}$$

Thanks to the presence of the other degrees of freedom, one might think that in the dominant configurations such singularities are actually regularized by the zero of $1 - |\phi|^2$.

Indeed, the standard Yang-Mills action written in these variables reads

$$\begin{aligned} S &= -\frac{1}{4g^2}\int d^4x\, F_{\mu\nu}^2 = -\frac{1}{4g^2}\int d^4x\, \{\, [G_{\mu\nu} + (1-|\phi|^2)\, H_{\mu\nu}]^2 + \\ &+ 2\,[\sum_{\mu\neq\nu} \partial_\mu \mathbf{n}\cdot\partial^\mu \mathbf{n}\, (D_\nu\phi)^*(D^\nu\phi) - \sum_{\mu\neq\nu} \partial_\mu \mathbf{n}\cdot\partial_\nu \mathbf{n}\, (D^\mu\phi)^*(D^\nu\phi)] \\ &- i\,[(D^\mu\phi)^*(D^\nu\phi) - (D^\nu\phi)^*(D^\mu\phi)]\, H_{\mu\nu}\,\} + \{\theta \text{ - term}\}, \end{aligned} \tag{4}$$

where [1]

$$G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu; \quad \phi = \rho + i\sigma, \quad H_{\mu\nu} = (\mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}); \tag{5}$$

$$D_\mu \phi = (\partial_\mu - iC_\mu)\phi. \tag{6}$$

In the case of the Wu-Yang monopole, the singularity in the energy at the origin comes from the behavior $H_{ij}^2 = 2/r^4$.

Note that the local $U(1)$ invariance (corresponding to $SU(2)$ gauge transformations of the form $U = \exp i\alpha\mathbf{n} \cdot \tau/2)$ is manifest in Eq.(4). Fixing the direction of $\mathbf{n}$ (by gauge transformations belonging to $SU(2)/U(1)$) as $\mathbf{n} = (0,0,1)$, for example, amounts to the Abelian gauge fixing: for the $\mathbf{n}$ configuration of the form, $n^a = \frac{x^a}{r}$, such a gauge transformation would introduce an (apparently) singular Dirac monopole, even though the gauge field configuration itself is perfectly regular.

A curious feature of Faddeev-Niemi decomposition, that the connection contains $\tilde{\sigma}(x) = 1 + \sigma(x)$ (and $\rho(x)$) naturally while the action depends on $\sigma(x)$ (and $\rho(x)$) in a simple and significant way, is central to our discussion.

Before going into our main argument, let us note, following Faddeev and Niemi, that the form of Eq.(4) suggests a very clear picture of different possible phases of QCD. Namely, if the field $\phi(x)$ fluctuates more strongly than the $\mathbf{n}$ field, one could integrate the former first, in the sence of renormalization group, arriving at a low-energy effective action of QCD,

$$S_{FN}^{eff} = \int d^4x \, \{\Lambda^2 \sum_\mu \partial_\mu \mathbf{n} \cdot \partial_\mu \mathbf{n} + \sum_{\mu \neq \nu} (\mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})^2\}, \tag{7}$$

suggested in [4], describing the confinement phase of QCD (Λ is a dynamically generaled mass). This action has two important features, one being the unique action containing $\mathbf{n}$ field and allowing for Hamiltonian interpretation, and second, containing topological solitons [15] which could be thought as models of gluonia. Though very interesting, it is not our main interest here to pursue these ideas further.

[1] The θ term has also quite an elegant form,

$$\begin{aligned} \frac{\theta}{32\pi^2} \int d^4x \, F_{\mu\nu} \tilde{F}^{\mu\nu} &= \frac{\theta}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} \int d^4x \, \{[G^{\mu\nu} + (1 - |\phi|^2) \, H^{\mu\nu}][G^{\rho\sigma} + (1 - |\phi|^2) \, H^{\rho\sigma}] \\ &+ \; i \, [(D^\mu \phi)^* (D^\nu \phi) - (D^\nu \phi)^* (D^\mu \phi)] \, H^{\rho\sigma}\}. \end{aligned}$$

Vice versa, if dynamically $\mathbf{n}$ field fluctuates more, renormalization group flow would instead yield a low energy action which looks more like

$$S_{FN}^{eff'} = -\int d^4x \{ G_{\mu\nu}^2 - (D_\nu \phi)^*(D_\nu \phi) + (1 - |\phi|^2)^2 \} \tag{8}$$

which is the standard Higgs model (describing a possible Higgs phase). Finally, if none of the fields fluctuate strongly, then one would have the original Yang-Mills action at low energies: one is in the non-Abelian Coulomb phase in this case (this will be the case if a sufficient number of massless fermions are present).

Though obviously over-simplified, these observations do show that field configurations which contribute to nonvanishing vev $\langle\phi\rangle \neq 0$ tend to bring the system to the Higgs phase (short-ranged color electric force); confinement is caused by configurations giving rise to dual Higgs phase (hence with $\langle\phi\rangle = 0$).

A zero-energy static left-handed quark field satisfies the equation (we shall consider the massless quarks for simplicity)

$$i(\sigma_k)_{\alpha\beta}[\partial_k - i(C_k n^a + \tilde{\sigma}(\partial_k \mathbf{n} \times \mathbf{n})^a + \rho\, \partial_k \mathbf{n}^a)\frac{\tau^a}{2}]_{ij}\, \psi_{L\beta j} = 0. \tag{9}$$

which follows from the standard covariant Weyl equation

$$i\bar{\sigma}_\mu(\partial_\mu - iA_\mu)\psi_L = 0, \tag{10}$$

with A_μ taken in the Faddeev-Niemi form. The exact $U(1)$ local invariance was used to set $C_0(x) = 0$ above.

As is well known the Dirac Hamiltonian (the operator multiplying ψ_L in Eq.(9)) commutes with the total "angular momentum" operator,

$$\mathbf{J} = \mathbf{L} + \mathbf{s} + \frac{\tau}{2}, \tag{11}$$

composed of the orbital and spin angular momenta *and* the gauge $SU(2)$ spin. For this reason one seeks for a singlet zero mode having the general structure

$$(\psi_L)_{\alpha i} = -i\tau_{\alpha i}^2\, g_L(r) + (\tau^a\tau^2)_{\alpha i}\, r^a h_L(r). \tag{12}$$

The functions $g_L(r)$ and $h_L(r)$ satisfy the coupled equations, ($j_L(r) \equiv r h_L(r)$):

$$\begin{aligned} j_L^{'} + \frac{2}{r} j_L - \frac{\tilde{\sigma}}{r} j_L + [\frac{r^2 C(r)}{2} + \frac{\rho}{r}]\, g_L &= 0; \\ g_L^{'} + \frac{\tilde{\sigma}}{r} g_L - [\frac{r^2 C(r)}{2} - \frac{\rho}{r}]\, j_L &= 0, \end{aligned} \tag{13}$$

where we have set

$$C_k(x) = r^k C(r), \tag{14}$$

and assumed spherically symmetric forms for $C(r)$, $\rho(r)$, and $\sigma(r)$.

Let us first consider the case of an instanton background at $x_0 = 0$,

$$n^a(x) = \frac{x^a}{r}; \qquad \tilde{\sigma}(r) = 2\,r^2\,f(x); \qquad f(x) = \frac{1}{r^2+\lambda^2}; \qquad C(r) = \rho(r) = 0, \tag{15}$$

where λ is the instanton size. This is actually only the instanton configuration at $x_0 = 0$, the static configuration relevant to the *three* dimensional zero mode. We shall loosely refer to it as the instanton below. Note that the monopole singularity at the origin is smoothened by the zero of $1 - |\phi|^2$ since $\tilde{\sigma}(x) \to 0$ (or $\sigma(x) \to -1$) as $r \to 0$ and $\rho \equiv 0$. Eq.(13) can be immediately integrated in this case and gives

$$g_L = e^{-\int_0^r dr\,\frac{\tilde{\sigma}}{r}} = \frac{1}{r^2+\lambda^2}; \qquad j_L = 0, \tag{16}$$

which is the well-known three-dimensional zero mode of the lefthanded fermion [16], related to the four-dimensional Euclidean zero mode by the spectral-flow argument [13]. The instanton background at $x_0 \neq 0$ does not allow for three-dimensional fermion zero modes.

Note also that there is another independent solution of Eq.(13)

$$g_L = 0; \quad j_L(r) = \exp - \int_0^r dr \frac{2-\tilde{\sigma}}{r}, \tag{17}$$

which is however not normalizable.

Of course, the precise form of the instanton background is not needed for the existence of the normalizable fermion zero mode. The essential features of the background (15) are:

i) the hedgehog form of the $\mathbf{n}$ field;

ii) the behavior of the $\sigma(r)$ field (in the gauge $rho = 0$),

$$\tilde{\sigma}(0) = 0; \qquad \tilde{\sigma}(r) \stackrel{r\to\infty}{\longrightarrow} 2 \quad (\text{or} \quad \sigma(r) \stackrel{r\to\infty}{\longrightarrow} 1). \tag{18}$$

In fact, the first of (18) guarantees that the gauge configuration is regular at the origin and that the zero mode is normalizable at the origin, while the second, which means that it looks like a monopole of charge 2 from the distance, leads to the asymptotic behavior of the fermion zero mode, $g_L \sim 1/r^2$, compatible with the normalizability at $r \to \infty$.

Another example of configuration of this class is the di-meron configuration [17, 16]

$$\begin{aligned} n^a(x) &= \frac{x^a}{r}; \qquad \tilde{\sigma}(r) = r\frac{\partial}{\partial r}R(x); \quad C(r) = \rho(r) = 0, \\ R(x) &= \log[r^2 + y^2 - t^2 + \{r^2 + (t-y)^2\}^{1/2}\{r^2 + (t+y)^2\}^{1/2}]; \end{aligned} \tag{19}$$

for $-y < t < y$ ($2y$ is the two-meron separation).

In fact we can consider a more general class of configurations with these characteristics, with possibly $C(r) \neq 0$, $\rho(r) \neq 0$, and call these *instanton-like configurations.* Although one has coupled equations for the system (g_L, j_L), one can quite generally assume that a normalizable solution (g_L, j_L) exists if $C(r)$ and ρ are sufficiently small.

The instanton-like configurations may then well have to do with the chiral symmetry breaking, since in the collection of such configurations ("instanton liquid" [18]) the chiral symmetry breaking vevs

$$\langle \bar{u}_R u_L \rangle = \langle \bar{d}_R d_L \rangle \neq 0 \tag{20}$$

will be nonvanishing.

Instanton-like configurations are on the other hand of no use from the point of view of confinement. In fact, the asymptotic behavior of the σ field means that $|\phi| \to 1$ at infinity, suggesting that instanton-like configurations tend to bring the system into the Higgs phase [2]. This is consistent with the general idea that the instantons, being point-like in four dimensions, have nothing to do with confinement, but our argument is based on the three dimensional properties of these configurations.

Consider next the class of configurations with the following characteristics:

i) the **n** field is the hedgehog form $n^a = x^a/r$;

ii) the $\sigma(r)$ field behaves as (we set $\rho = 0$ by the $U(1)$ gauge transformation),

$$\tilde{\sigma}(0) = 0; \qquad \tilde{\sigma}(r) \overset{r\to\infty}{\longrightarrow} 1, \tag{21}$$

namely

$$A_i^a = \tilde{\sigma}(x)\, \epsilon_{aij} \frac{x^j}{r^2} + \ldots \tag{22}$$

[2]See [22] for recent related ideas.

which we call *regularized Wu-Yang monopole* configurations. The difference of factor 2 in the asymptotic behavior of $\tilde{\sigma}(x)$ field as compared to the instanton-like configurations, is crucial. It means that, on the one hand, $\sigma(r) \to 0$ ($\phi \to 0$) at infinity so these configurations are consistent with confinement; on the other hand, it implies that the fermion zero modes are non-normalizable (see below).

Note that the single meron configuration at a fixed time slice

$$
\begin{aligned}
n^a(x) &= \frac{x^a}{r}; \qquad \tilde{\sigma}(r) = r\frac{\partial}{\partial r}R(x); \quad C(r) = \rho(r) = 0, \\
R(x) &= \log[\{r^2 + t^2\}^{1/2} - t],
\end{aligned} \tag{23}
$$

is of this type: in fact this class of configurations may alternatively be called meron-like configurations.

If (collection of) these configurations are indeed dominant in the ground state of QCD, it will lead to dual Meissner effect (confinement), as suggested by 't Hooft, Mandelstam and Nambu.

These configurations however do not trigger chiral symmetry breaking, since there are no normalizable fermion zero modes in this case. In fact if we assume $C(r) = 0$ for simplicity, one of the solutions of the zero-energy equation is again

$$
g_L = e^{-\int_0^r dr\, \frac{\tilde{\sigma}}{r}} \tag{24}
$$

but this is not normalizable at infinity, because it behaves as $g_L \sim 1/r$. In the case of the meron configuration this solution coincides with the one given in [16]. The other solution $j_L(r)$ (Eq.(17)) is non-normalizable in this case, too. Again, we could allow for nonvanishing $C(r)$ and $\rho(r)$ fields, but it is clear that for quite general class of perturbations these solutions will remain both non-normalizable.

We conclude that regularized Wu-Yang monopoles (or rather collections of those) are fundamental for confinement but have in themselves nothing to do with chiral symmetry breaking. It is quite remarkable that one can identify, through the analysis of semiclassical fermion zero modes, two distinct and complementary sets of configurations: one (*instanton-like configurations*) has likely to do with chiral symmetry breaking but with no relation to confinement and the other (*regularized Wu-Yang monopoles*) being most likely responsible for confinement but are, as they are, unrelated to chiral symmetry breaking. This suggests that the two main nonperturbative effects of QCD, confinement and chiral symmetry breaking, are distinct effects, and in particuar that the latter (chiral symmetry breaking) is not a direct consequence of the former (confinement). There are hints that support this conclusion in the lattice approach to QCD [19].

In fact, our conclusion overlaps considerably with that of Callan et. al. [13], but the use of the Faddeev-Niemi decomposition appears to allow for a particularly simple way to relate the question of chiral symmetry breaking (existence or absence of the fermion zero modes) to that of confinement.

We conclude with several comments.

1. The fact that the asymptotic value of σ is quantized (in the gauge $\rho(x) = 0$),

$$\sigma(r) \stackrel{r\to\infty}{\longrightarrow} n, \quad n \in \mathbf{Z}, \tag{25}$$

   is fundamental to our discussion. (25) must be imposed on the Faddeev-Niemi construction. This is so because in the gauge $\mathbf{n} = (0, 0, 1)$ and for $\rho = 0$, $\tilde{\sigma}(\infty)$ represents the charge of the magnetic monopole (see Eq.(22)), which should obey Dirac's quantization condition for consistency if fermions are present in the theory. It should be noted that although $\mathbf{n}$ field represents $\Pi_2(SU(2)/U(1))$ hence can be devided into integer classes of winding number $S^2 \to S^2$, this fact alone is not sufficient to guarantee the integer monopole charge.

2. Our analysis seems to shed some new light on the long-standing question of the interplay between the instantons and the monopoles in QCD. Although they are clearly not totally unrelated, their relations [20] can be rather subtle. For instance, in the exact Seiberg-Witten solution [8] of $N = 2$ supersymmetric gauge theories, the renormalization of the θ parameter is due to the infinite sum of instanton contributions, but in the dual language more adequate at low energies it is seen as due to the perturbative loops of magnetic monopoles. See also an earlier related idea [21] and recent related results in finite temperature QCD [22].

3. It is known that confinement can occur without chiral symmetry breaking, as is exemplified in many supersymmetric models, supporting our idea that these two phenomena are in principle distinct. For example, in the massless $N = 1$ supersymmetric QCD with $n_f = n_c + 1$, the low-energy degrees of freedom are mesons and baryons and their superpartners (confinement), while chiral symmetries remain unbroken in one of the possible vacua [23]. It is suggestive

that this occurs precisely in a theory in which the effects of instantons are known to be relatively weak [23].

4. The work of Faddeev and Niemi generalizes an earlier proposal by Cho [24], who required that the gauge connection satisfy

$$D_\mu \mathbf{n} = 0, \tag{26}$$

whose solution is simply

$$A_\mu^a(x) = C_\mu n^a + (\partial_\mu \mathbf{n} \times \mathbf{n})^a. \tag{27}$$

The field tensor decomposes as a sum of "electric" and "magnetic" parts,

$$F_{\mu\nu} = G_{\mu\nu} + H_{\mu\nu}; \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu; \quad H_{\mu\nu} = (\mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}), \tag{28}$$

where $H_{\mu\nu}$ contains the (Wu-Yang) magnetic monopole for $\mathbf{n}$ field of hedgehog form. The decomposition by Faddeev and Niemi continues to share this nice property, but having extra degrees of freedom ($\phi(x)$) is capable of accomodating larger classes of regular field configurations which are needed in our discussion.

5. Actually, the decomposition of the $SU(2)$ connection used here is not a fully general one. $A_\mu^a(x)$ is expressed in terms of six independent functions (after taking into account of the local $U(1)$ invariance, one has two physical degrees of freedom in C_μ, two for $\mathbf{n}$ and two more from $\phi(x)$), which matches the number of the physical degrees of freedom of $A_\mu^a(x)$. They were shown to be complete [4] in the sense that Yang-Mills equations of motions are correctly reproduced by those for $C_\mu(x), \mathbf{n}(x), \phi(x)$. Of course, as functional integration variables one needs two more degrees of freedom, and the Faddeev-Niemi decomposition has been accordingly generalized recently [25]. The original "on-shell" decomposition, however, seems to contain just sufficient number of degrees of freedom (i.e., a just sufficiently wide classes of configurations) for the purpose of the present exploratory study.

6. Very recently Davies et.al.[26] have computed the gluino condensate in pure $N = 1$ supersymmetric Yang-Mills theory, by compactifying the time coordinate on a cylinder, and by using the gluino zero mode in the background of the standard BPS monopole for $A_i^a(x)$ ($A_0^a(x)$ appears as the Higgs scalar). One might wonder whether the absence of fermion zero modes in the background of the regularized

Wu-Yang monopole we noted, is compatible with their work. In fact, if one repeats our analysis in the case of a fermion in the adjoint representation of the gauge group, one does find a pair of zero modes in the background of regularized Wu-Yang monopole background. The simplest way to get this result is to set the Yukawa coupling to zero in the original analysis of Jackiw and Rebbi [7]. In the case of the fundamental fermion, it can be seen that the normalizable Jackiw-Rebbi zero mode becomes non-normalizable [3], which is compatible with the absence of normalizable zero modes we found here. On the other hand, the (pair of) normalizable zero modes found in [7] continue to be normalizable in the limit of the vanishing Yukawa coupling in the case of an adjoint fermion [4] which is consistent with the calculation of [26].

7. In an alternative QCD with quarks in the adjoint representation, only the regularized Wu-Yang configurations would lead to confinement while chiral symmetry breaking can be caused by both types of configurations. It is thus possible that at some finite temperature chiral symmetry breaking can occur without confinement, as suggested by Kogut et. al. [19], if instanton-like configurations dominate over Wu-Yang monopole like configurations.

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